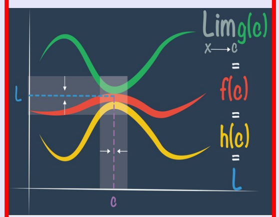
Calculus I

Lecture 5



Class Quiz 5

$\boxed{\text{Box Your Final Answer}}$

Evaluate the following limits:

1) $\lim_{x\to 4} \dfrac{x-4}{2-\sqrt{x}} = \dfrac{4-4}{2-\sqrt{4}} = \dfrac{0}{2-2} = \dfrac{0}{0}$ I.F.

$= \lim_{x\to 4} \dfrac{(x-4)(2+\sqrt{x})}{(2-\sqrt{x})(2+\sqrt{x})}$

$= \lim_{x\to 4} \dfrac{\overset{-1}{\cancel{(x-4)}}(2+\sqrt{x})}{\cancel{4-x}}$

$= \lim_{x\to 4} -(2+\sqrt{x}) = -(2+\sqrt{4})$

$= \boxed{-4}$

2) $\lim_{x\to -1} \dfrac{2x^2-3x-5}{x^2+3x+2}$

$= \dfrac{2(-1)^2-3(-1)-5}{(-1)^2+3(-1)+2} = \dfrac{0}{0}$ I.F.

$= \lim_{x\to -1} \dfrac{\cancel{(x+1)}(2x-5)}{\cancel{(x+1)}(x+2)}$

$= \lim_{x\to -1} \dfrac{2x-5}{x+2} = \dfrac{2(-1)-5}{-1+2} = \dfrac{-7}{1}$

$= \boxed{-7}$

Evaluate

1) $\lim_{x \to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x} = \dfrac{\frac{1}{4}+\frac{1}{-4}}{4+(-4)} = \dfrac{0}{0}$ I.F.

LCD $= 4x$

$= \lim_{x \to -4} \dfrac{4x \cdot \frac{1}{4} + 4x \cdot \frac{1}{x}}{4x(4+x)} = \lim_{x \to -4} \dfrac{\cancel{x+4}}{4x\cancel{(4+x)}} = \lim_{x \to -4} \dfrac{1}{4x}$

$= \dfrac{1}{4(-4)} = \boxed{\dfrac{-1}{16}}$

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2) $\lim_{h \to 0} \dfrac{(3+h)^{-1}-3^{-1}}{h} = \dfrac{(3+0)^{-1}-3^{-1}}{0} = \dfrac{3^{-1}-3^{-1}}{0} = \dfrac{0}{0}$ I.F.

$= \lim_{h \to 0} \dfrac{\frac{1}{(3+h)^1} - \frac{1}{3^1}}{h}$

$x^{-n} = \dfrac{1}{x^n}$

$x^1 = x$

$= \lim_{h \to 0} \dfrac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h \to 0} \dfrac{3-(3+h)}{h(3+h)\cdot 3}$

LCD $= (3+h)\,3$

$= \lim_{h \to 0} \dfrac{\cancel{3}-\cancel{3}-\overset{-1}{\cancel{h}}}{\cancel{h}(3+h)\cdot 3}$

$= \lim_{h \to 0} \dfrac{-1}{(3+h)\cdot 3} = \dfrac{-1}{(3+0)\cdot 3}$

$= \boxed{\dfrac{-1}{9}}$

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Remember

$$\boxed{\lim_{h\to 0}\frac{\sin h}{h}=1} \quad \& \quad \boxed{\lim_{h\to 0}\frac{1-\cos h}{h}=0}$$

Evaluate

1) $\lim_{x\to 0}\frac{\sin 3x}{5x} = \lim_{x\to 0}\frac{3\sin 3x}{3\cdot 5x} = \lim_{x\to 0}\left(\frac{3}{5}\cdot\frac{\sin 3x}{3x}\right)$

$=\frac{\sin 0}{0}=\frac{0}{0}$ I.F.

$=\lim_{x\to 0}\frac{3}{5}\cdot\lim_{x\to 0}\frac{\sin 3x}{3x} = \frac{3}{5}\cdot 1 = \boxed{\frac{3}{5}}$

2) $\lim_{x\to 0}\frac{\sin 3x}{\sin 5x} = \frac{\sin 0}{\sin 0} = \frac{0}{0}$ I.F.

$=\lim_{x\to 0}\frac{3\frac{\sin 3x}{3x}}{5\frac{\sin 5x}{5x}} = \frac{3}{5}\cdot\frac{\lim_{x\to 0}\frac{\sin 3x}{3x}}{\lim_{x\to 0}\frac{\sin 5x}{5x}} = \frac{3}{5}\cdot\frac{1}{1} = \boxed{\frac{3}{5}}$

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3) $\lim_{x\to 1}\frac{\sin(x-1)}{x^2+x-2} = \frac{\sin(1-1)}{1^2+1-2} = \frac{\sin 0}{0} = \frac{0}{0}$ I.F.

$=\lim_{x\to 1}\frac{\sin(x-1)}{(x-1)(x+2)} = \lim_{x\to 1}\left[\frac{\sin(x-1)}{x-1}\cdot\frac{1}{x+2}\right]$

$=\boxed{\lim_{x\to 1}\frac{\sin(x-1)}{x-1}}\cdot\boxed{\lim_{x\to 1}\frac{1}{x+2}}$

$=1\cdot\frac{1}{1+2} = 1\cdot\frac{1}{3} = \boxed{\frac{1}{3}}$

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Evaluate $\dfrac{\sin 3x \sin 5x}{x^2}$

Make sure you are in radian mode.

1) when $x = .0001$

$$\frac{\sin .0003 \cdot \sin .0005}{.0001^2} \approx 14.99999915\ldots$$

2) when $x = -.0001$

$$\frac{\sin(-.0003)\cdot\sin(-.0005)}{(-.0001)^2} \approx 14.99999915\ldots$$

$\sin(-x) = -\sin x$

Evaluate

$$\lim_{x\to 0}\frac{\sin 3x \cdot \sin 5x}{x^2} = \frac{\sin 0 \cdot \sin 0}{0^2} = \frac{0}{0} \quad \text{I.F.}$$

$$= \lim_{x\to 0}\left[\frac{3\sin 3x}{3x}\cdot\frac{5\sin 5x}{5x}\right]$$

$$= 3\cdot \cancel{\lim_{x\to 0}\frac{\sin 3x}{3x}}^{\,1} \cdot 5 \cdot \cancel{\lim_{x\to 0}\frac{\sin 5x}{5x}}^{\,1} = 3\cdot 1\cdot 5\cdot 1$$

$$= \boxed{15}\ \checkmark$$

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Evaluate

$$\lim_{h\to 0}\frac{1-\cos h}{\sin h} = \frac{1-\cancel{\cos 0}^{\,1}}{\cancel{\sin 0}^{\,0}} = \frac{1-1}{0} = \frac{0}{0} \quad \text{I.F.}$$

$$= \lim_{h\to 0}\frac{\dfrac{1-\cos h}{h}}{\dfrac{\sin h}{h}} = \frac{\cancel{\lim_{h\to 0}\dfrac{1-\cos h}{h}}^{\,0}}{\cancel{\lim_{h\to 0}\dfrac{\sin h}{h}}^{\,1}} = \frac{0}{1} = \boxed{0}$$

Feb 26-9:57 AM

Suppose $2x \leq f(x) \leq x^4 - x^2 + 2$

evaluate $\lim_{x \to 1} f(x)$

Since $f(x)$ is between two other function

$\Rightarrow$ Try Squeeze thrm

$\lim_{x \to 1} 2x = 2(1) = \boxed{2}$ ✓

$\lim_{x \to 1} (x^4 - x^2 + 2) = 1^4 - 1^2 + 2 = \boxed{2}$ ✓

Subtraction

by S.T., $\lim_{x \to 1} f(x) = \boxed{2}$

Feb 26-10:03 AM

Precise Def. of limits:

For any $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$.

$$\lim_{x \to a} f(x) = L$$

Prove $\lim_{x \to 1} (4x - 2) = 2$

$a = 1$,
$f(x) = 4x - 2$
$L = 2$

1) verify the limit.

$\lim_{x \to 1} (4x - 2) = 4(1) - 2 = 4 - 2 = \boxed{2}$ ✓

2) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|4x - 2 - 2| < \varepsilon$ whenever $\boxed{|x - 1|} < \delta$

$|4x - 4| < \varepsilon$

$|ab| = |a||b|$

$|4(x - 1)| < \varepsilon$

$|4|\,|x - 1| < \varepsilon$

$4\,|x - 1| < \varepsilon$

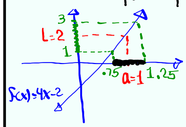
Divide by 4

$\boxed{|x - 1|} < \frac{\varepsilon}{4}$

Pick $\delta = \frac{\varepsilon}{4}$

If $\varepsilon = 1 \rightarrow \delta = \frac{1}{4} = .25$

If $\varepsilon = 2 \rightarrow \delta = \frac{2}{4} = .5$



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Prove $\lim_{x \to 4} \left(\frac{1}{2}x + 3\right) = 5$

$f(x) = \frac{1}{2}x + 3$ $\quad L = 5 \checkmark$ $\quad a = 4$

1) verify the limit

$\lim_{x \to 4} \left(\frac{1}{2}x + 3\right) = \frac{1}{2}(4) + 3 = 2 + 3 = \boxed{5} \checkmark$

2) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{2}x + 3 - 5\right| < \varepsilon$ 〃 $|x - 4| < \delta$

$\left|\frac{1}{2}x - 2\right| < \varepsilon$ 〃 $|x - 4| < \delta$

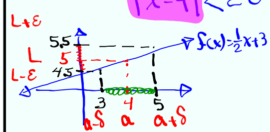
$\left|\frac{1}{2}(x - 4)\right| < \varepsilon$ 〃 $|x - 4| < \delta$

$\left|\frac{1}{2}\right| |x - 4| < \varepsilon$ 〃 $|x - 4| < \delta$

$\frac{1}{2}|x - 4| < \varepsilon$ 〃 $|x - 4| < \delta$

Multiply by 2

$|x - 4| < 2\varepsilon$ 〃 $|x - 4| < \delta$



$\boxed{\delta = 2\varepsilon}$

If $\varepsilon = .5 \to \delta = 1$

If $\varepsilon = .25 \to \delta = .5$

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Prove $\lim_{x \to 2} 4x = 8$

1) $f(x) = 4x$ $\quad L = 8 \checkmark$ $\quad a = 2$

2) Verify the limit

$\lim_{x \to 2} 4x = 4(2) = 8 \checkmark$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|4x - 8| < \varepsilon$ 〃 $|x - 2| < \delta$

$|4(x - 2)| < \varepsilon$ 〃 $|x - 2| < \delta$

$4|x - 2| < \varepsilon$ 〃 $|x - 2| < \delta$

Divide by 4

$|x - 2| < \frac{\varepsilon}{4}$ 〃 $|x - 2| < \delta$

Pick $\delta = \frac{\varepsilon}{4}$

Feb 26-10:52 AM

Prove $\lim_{x \to -2} \left(\frac{-1}{2}x + 6\right) = 7$

1) $f(x) = \frac{-1}{2}x + 6$ $\quad L = 7 \checkmark$ $\quad a = -2$

2) verify the limit.

$$\lim_{x \to -2} \left(\frac{-1}{2}x + 6\right) = \frac{-1}{2}(-2) + 6 = 1 + 6 = 7 \checkmark$$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{-1}{2}x + 6 - 7\right| < \varepsilon$ " $|x - (-2)| < \delta$

$\left|\frac{-1}{2}x - 1\right| < \varepsilon$ " $|x + 2| < \delta$

$\underbrace{\left|\frac{-1}{2}(x + 2)\right|} < \varepsilon$ " $|x + 2| < \delta$

$\left|\frac{-1}{2}\right| |x + 2| < \varepsilon$ " $|x + 2| < \delta$

$\frac{1}{2}|x + 2| < \varepsilon$ " $|x + 2| < \delta$

Multiply by 2

$|x + 2| < 2\varepsilon$ " $|x + 2| < \delta$

Pick $\boxed{\delta = 2\varepsilon}$

Feb 26-10:58 AM

Prove $\lim_{x \to 0} x^2 = 0$

1) $f(x) = x^2$ $\quad L = 0 \checkmark$ $\quad a = 0$

2) Verify the limit.

$$\lim_{x \to 0} x^2 = 0^2 = 0 \checkmark$$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 - 0| < \varepsilon$ " $|x - 0| < \delta$

$|x^2| < \varepsilon$ " $|x| < \delta$

Take square root of both sides

$\sqrt{|x^2|} < \sqrt{\varepsilon}$

$|x| < \sqrt{\varepsilon}$ whenever $|x| < \delta$

Pick $\boxed{\delta = \sqrt{\varepsilon}}$

Feb 26-11:06 AM

Prove $\lim_{x \to 3} x^2 = 9$

1) $f(x) = x^2$ $\quad L = 9 \checkmark$ $\quad a = 3$

2) Verify the limit.

$$\lim_{x \to 3} x^2 = 3^2 = 9 \checkmark$$

3) $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|x^2 - 9| < \varepsilon$ 〃 $|x-3| < \delta$

$|(x+3)(x-3)| < \varepsilon$ 〃 $|x-3| < \delta$

$|ab| = |a|\,|b|$

$\underbrace{|x+3|}_{\text{Bound}}\ \underbrace{|x-3|}_{\text{Keep}} < \varepsilon$ 〃 $|x-3| < \delta$

Suppose $\delta \le 1$ $\quad |x-3| < 1$

$-1 < x-3 < 1$

Add 6

$-1+6 < x-3+6 < 1+6$

$5 < x+3 < 7$

So $|x+3| < 7$

we had

$|x+3|\,|x-3| < 7\,|x-3| < \varepsilon$

$|x-3| < \frac{\varepsilon}{7}$

$$\boxed{\text{Pick } \delta = \min\left\{1, \frac{\varepsilon}{7}\right\}}$$

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